

Big Bang Nucleosynthesis Constraints on the Self-Gravity of Pressure

Saul Rappaport, Josiah Schwab, and Scott Burles

Department of Physics and Kavli Institute for Astrophysics and Space Research, MIT, Cambridge, MA 02139

Gary Steigman

*Departments of Physics and Astronomy and Center for Cosmology and Astro-Particle Physics,
The Ohio State University, 191 West Woodruff Ave., Columbus, OH 43210*

Using big bang nucleosynthesis and present, high-precision measurements of light element abundances, we constrain the self-gravity of radiation pressure in the early universe. The self-gravity of pressure is strictly non-Newtonian, and thus the constraints we set provide a direct test of this prediction of general relativity and of the standard, Robertson-Walker-Friedmann cosmology.

I. INTRODUCTION

Certain aspects of general relativity are well tested. For example, the Schwarzschild metric has been quantitatively verified in the weak-field limit on small scales, e.g., the Solar system [1, 2] and binary radio pulsars [e.g., 3, 4, 5]; and on galaxy scales [e.g., 6]. In another fundamental test of general relativity, the existence of gravity waves has been established [e.g., 7, 5]. General relativity theory, utilizing the Robertson-Walker metric [8, 9, 10] leads to the Friedmann equations [8, 11] which govern the expansion behavior of a homogeneous, isotropic Universe. However, it is probably fair to say that the Friedmann equations, while providing a self-consistent and highly successful framework for cosmology, have not been subjected to extensive, independent testing. In this paper we show that one particular aspect of the Friedmann equations, the self-gravity of pressure, can be tested quantitatively.

The development of big-bang nucleosynthesis (BBN) codes [12, 13], coupled with measurements of the relevant nuclear reaction rates [14, 15], have allowed observations of light element abundances to become powerful tools with which to investigate the early evolution of the universe. Computational predictions over a wide range of parameter space, when compared with primordial abundances inferred from observations, have yielded constraints on the current-epoch baryon density [16, 17, 18, 19], neutrino physics [17, 18, 19, 20], the fine structure constant [21], the gravitational constant [17, 19, 22, 23], primordial magnetic fields [24], the universal lepton asymmetry [12, 18, 19] and other parameters of astrophysical interest.

Increasingly accurate measurements of element abundances, as well as improved understanding of the processes (i.e., stellar and galactic nucleosynthesis) which have altered the original abundances, allow these restrictions to be continually refined. Deuterium abundances [25, 26], helium abundances [27, 28, 29], and lithium abundances [30, 31] have all been well measured, although the inferred primordial abundances are subject to large and often difficult to quantify systematic uncertainties. More recently, observations of the cosmic microwave background (CMB) have yielded an indepen-

dent estimate of η , the baryon to photon ratio at a much later epoch in the evolution of the Universe [32].

II. ANALYSIS

A. Friedmann Equations

For an isotropically expanding universe in which the matter/energy is distributed homogeneously, the expansion of the universe is described by a time-dependent scale factor, $a = a(t)$. In the standard Friedmann-Robertson-Walker (FRW) cosmology, the time variation of the scale factor is given by the Friedmann equations in terms of the average density and pressure. For example, the “acceleration” of the scale factor, \ddot{a} , is given by:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2} \right), \quad (1)$$

where ρc^2 is the energy density and P is the pressure. This is the exact \mathcal{G}_{rr} component of the Einstein field equation for a homogenous and isotropic universe. Note that the $3P$ term, implying the self-gravity of pressure, is a purely General Relativistic (GR) effect, with no analog in Newtonian gravity. The “velocity” of the scale factor, \dot{a} , is given by the Friedmann-Lemaître equation:

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{kc^2}{a^2}, \quad (2)$$

whose origin is the \mathcal{G}_{tt} component of the Einstein field equations (the second term on the right hand side of the equation is due to the curvature; k is the curvature constant which appears in the Robertson-Walker metric). For any fluid, given its equation of state, i.e., $P = P(\rho)$, eq. (1) can be integrated to yield eq. (2) – but only if the $3P$ term is included.

We would now like to test the Friedmann-Lemaître equations by placing constraints on the existence of the $3P/c^2$ term in eq. (1), and to see what the testable consequences are for eq. (2). To do this we will, by necessity, no longer be assuming the validity of GR. However, we will retain the energy conservation of expanding fluids

via the first law of thermodynamics (the perfect fluid approximation or entropy conservation).

We start with a “Newtonian cosmology” [33, 34] which, of course, cannot be completely justified outside the context of GR, but which nonetheless provides considerable insight into our testing of the $3P/c^2$ term [35]. For the usual Newtonian gravity, this amounts to

$$\frac{\ddot{a}}{a} = - \left(\frac{4\pi}{3} \right) G \rho . \quad (3)$$

For the special, zero-pressure case where $\rho = \rho_0 a^{-3}$, eq. (3) can be integrated to yield the familiar expression for \dot{a} :

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\text{constant}}{a^2} . \quad (4)$$

For the Newtonian analysis, the constant in eq. (4) is simply a constant of integration, in contrast to the curvature term which appears in the Friedmann equation, eq. (2).

This form of eq. (4) is, however, only valid for the special case of a pressureless fluid. What is missing from eq. (3) for the general case of a fluid with non-zero pressure is a term accounting for the self-gravity of pressure (see eq. 1), which has no expression in a purely Newtonian formulation. Suppose we now add such a term to eq. (3), in an *ad hoc* fashion, with an arbitrary multiplicative constant, χ .

$$\left(\frac{\ddot{a}}{a} \right) = - \left(\frac{4\pi}{3} \right) G \rho [1 + \chi(3P/\rho c^2)] . \quad (5)$$

For $\chi = 1$ we incorporate the full effect of the self-gravity of pressure (as it follows from GR; eq. 1), while for $\chi = 0$, this non-Newtonian effect is completely neglected. This is our proposed modification of the Friedmann equation,

eq. (1). Using the first law of thermodynamics for an adiabatic expansion expressed as:

$$d(\rho c^2 a^3) = -P d(a^3) , \quad (6)$$

we can solve eq. (6) for \dot{a}^2 :

$$\dot{a}^2 = -\frac{8\pi G}{3} \left\{ (1 - 3\chi) \int \rho a da - \chi \int a^2 d\rho \right\} . \quad (7)$$

Note that only for $\chi = 1$ (i.e., the full implementation of the pressure self-gravity term) is the standard form of the \dot{a}^2 version of the Friedmann equation recovered, viz,

$$\dot{a}^2 = \frac{8\pi G}{3} \int d(\rho a^2) = \frac{8\pi G}{3} \rho a^2 + \text{constant} . \quad (8)$$

For any equation of state of the form $P = w\rho c^2$ where w is a constant, the integrals in eq. (8) yield Friedmann-like equations, but with a modified leading coefficient:

$$\left(\frac{\dot{a}}{a} \right)^2 = H^2 = \left[\frac{1 + 3w\chi}{1 + 3w} \right] \frac{8\pi G}{3} \rho + \frac{\text{constant}}{a^2} . \quad (9)$$

For a matter-, radiation-, or vacuum energy-dominated universe, $3w = 3P/\rho c^2 = 0, 1$, and -3 , respectively. If $\chi = 1$, the proper Friedmann equation is recovered, regardless of the choice of equation of state. If $\chi = 0$, for a pressureless gas (“matter”), the correct Friedmann result is recovered; but, as soon as the expanding fluid has significant pressure, an incorrect result (with respect to GR) is obtained. Having set aside GR, which would otherwise connect the integration constant in equations (8) & (9) to the geometry, there is no fundamental connection to the geometry of the underlying space-time in our proposed modification of the Friedmann equations.

For the case where a combination of radiation, matter, and vacuum densities are considered together, eq. (9) can be written in a nearly familiar form:

$$\left(\frac{\dot{a}}{a} \right)^2 = H^2 = H_0^2 \left[\frac{\Omega_M}{a^3} + \frac{\Omega_R}{a^4} \left(\frac{1 + \chi}{2} \right) + \Omega_V \left(\frac{3\chi - 1}{2} \right) + \frac{\Omega_k}{a^2} \right] , \quad (10)$$

where the Ω ’s are defined by $\Omega_j = 8\pi G \rho_{j,0} / 3H_0^2$, and the $\rho_{j,0}$ ’s are, in turn, the densities of the respective constituents evaluated at the current epoch. Finally, in this expression $\Omega_k \equiv 1 - [\Omega_M + (\frac{1+\chi}{2}) \Omega_R + (\frac{3\chi-1}{2}) \Omega_V]$. Here we reemphasize that Ω_k can no longer be interpreted in terms of the curvature – it is just a constant of integration. If $\chi \neq 1$ eq. (10) has consequences for the CMB and current matter/vacuum dominated epochs, in addition to the BBN epoch, some of which could independently constrain χ , but these are beyond the scope of the present work.

We now focus on the epoch when the energy density

contributed by relativistic particles (including relativistic neutrinos), “radiation”, dominated the energy density of the Universe and radiation pressure is important. Our goal is to constrain χ by comparing BBN predictions with observations of light element abundances. The expansion rate of the Universe during this epoch is described by

$$\left(\frac{\dot{a}}{a} \right)^2 = \left(\frac{1 + \chi}{2} \right) \frac{8\pi G}{3} \rho_R , \quad (11)$$

where the Ω_k/a^2 term which appears in eq. (10) has been dropped because it is negligible compared to the Ω_R/a^4

term during the radiation-dominated, BBN epoch. As revealed by eq. (11), the effect of a value of χ which differs from unity is to change the early-universe expansion rate (Hubble parameter) from its standard value. In this sense, $\chi \neq 1$ is equivalent to an early-universe value of the gravitational constant [22] which differs from its present value or, a total relativistic energy density which differs from its standard-model value (as often parameterized by the effective number of neutrinos: $\rho'_R/\rho_R \equiv 1 + 7\Delta N_\nu/43$) [20].

B. Nucleosynthesis Calculations

Nucleosynthesis calculations were performed with a BBN code which has been updated with the latest reaction rates and whose output has been compared to that of other, published codes. Since the parameter we seek to constrain, i.e., $(1 + \chi)/2$, is multiplicative with G , we have simply varied G as a surrogate for χ . Thus, in our case the BBN-predicted abundances are functions of the baryon density parameter $\eta_{10} = 10^{10}\eta \equiv 10^{10}(n_B/n_\gamma)$ and G . In this work ~ 200000 BBN calculations were performed, varying χ from 0 to 2 in steps of 0.002 (or equivalently G/G_0 from 1/2 to 3/2 in linear steps of 0.001) and varying $\log \eta$ from -10 to -9 in steps of $1/200$ dex. The results of these calculations are the isoabundance contours for deuterium, helium-4, and lithium-7 shown in the $\{\eta, \chi\}$ plane in Figure 1. As estimated in [36], for fixed values of η_{10} and G , the uncertainties in the nuclear reaction rates contribute a $\sim 3\%$ uncertainty ($\sim 1\sigma$) to the BBN-predicted abundance of deuterium, and as estimated in [37] $\gtrsim 0.2\%$ ($\sim 1\sigma$) for the ^4He mass fraction.

C. Light Element Observations

Deuterium provides an excellent constraint on the baryon density because its post-BBN evolution is simple (D is only destroyed when gas is cycled through stars) and the observed amounts require that it must have formed in the big bang rather than in stellar or galactic processes [39, 40]. Also, its BBN-predicted abundance is extremely sensitive to the baryon to photon ratio, $D/H \propto \eta^{-1.6}$ [17, 37]. Deuterium measurements along the lines of sight to high redshift quasars have led to the current determination of $\log(D/H)_P = -4.55 \pm 0.04$ [1σ confidence; 26]. Contours of constant deuterium abundance (by number) are shown as green curves in Fig. 1.

Although BBN production of ^4He is relatively insensitive to η , its abundance provides an extremely useful constraint on the early-universe expansion rate (the Hubble parameter) and, therefore, on χ . As stars and galaxies evolve, stellar nucleosynthesis results in some post-BBN production of ^4He . As a consequence, the primordial abundance (mass fraction) of ^4He , Y_P , is best determined from present-day observations of low-metallicity, extragalactic HII regions which are less contaminated by

post-BBN produced ^4He . Since the total number of such HII regions exceeds 80 [28], it is not surprising that the formal, statistical uncertainty in Y_P is small. However, it has been well known for decades [41] that systematic corrections, such as underlying stellar absorption, ionization corrections, collisional excitations, etc., have the potential to change the central value of Y_P as well as to increase significantly the error budget. The largest data set of consistently observed and analyzed HII regions is from Izotov and Thuan [28] who find $Y_P = 0.243 \pm 0.001$. Izotov et al. have recently revised this to 0.247 ± 0.001 [1σ confidence; 29]. These analyses largely ignore most sources of systematic uncertainty, resulting in an error, largely statistical, which is too small to reflect the true uncertainty in Y_P . Accounting for some, but not all sources of systematics, and employing a model-dependent linear extrapolation of Y to zero oxygen abundance, Y_P has very recently been inferred by Peimbert et al. [38] to be 0.248 ± 0.003 . Contours of constant helium-4 mass fraction are shown as blue curves in Fig. 1.

It is interesting to note that for SBBN ($\chi = 1$), as well as for BBN with χ allowed to be free, the predicted primordial abundances of deuterium and lithium are strongly coupled [18]; see, Fig. 1. For our choice of the primordial D abundance, and for either choice of the primordial ^4He abundance, the predicted primordial lithium abundance lies in the range $12 + \log(\text{Li}/\text{H}) = 2.6 - 2.7$. This is in contrast to the best determinations of the lithium abundance in the oldest, most metal-poor stars in the halo of the Galaxy, where $12 + \log(\text{Li}/\text{H}) \simeq 2.1$ [30, 31]. The generally accepted explanation of this factor of 3 – 4 discrepancy is that the lithium observed at present in these oldest stars in the Galaxy has been diluted/depleted from the initial lithium abundance in the gas out of which these, nearly primordial, stars formed [42, 43] but, for a contrary point of view, see Bonifacio et al. [44].

D. BBN Constraints on χ

Figure 1 displays the results of our analysis. A section of the $\{\eta, \chi\}$ parameter space is shown, with *number* density (relative to hydrogen) contours for deuterium shown in green, for lithium shown in purple, and for the *mass* fraction of He-4 shown in blue. Notice that the pairs of $\{D/H, Y_P\}$ or of $\{\text{Li}/\text{H}, Y_P\}$ abundances form nearly orthogonal grids in the $\{\eta, \chi\}$ plane, so that the primordial abundances of either pair of these nuclides are sufficient to bound the cosmologically interesting parameters χ and the baryon density parameter η_{10} . Given the uncertainty in inferring the primordial lithium abundances from the observational data, only the deuterium and helium-4 pair is used in our analysis.

Assuming statistically-independent Gaussian errors (almost certainly, neither the errors in D nor those in ^4He are truly Gaussian), one can calculate the probability, via a maximum likelihood analysis, that the abun-

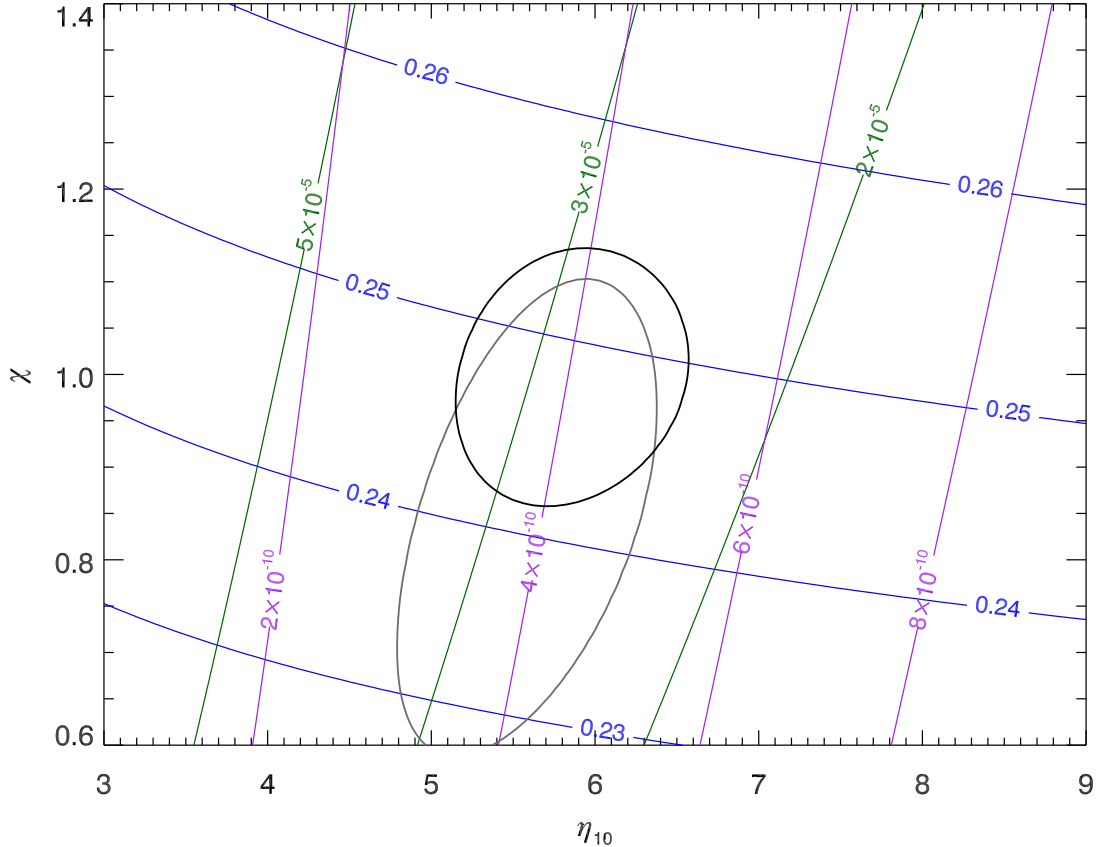


FIG. 1: Isoabundance contours for the predicted primordial abundances of deuterium and helium-4 in the $\chi - \eta_{10}$ plane, along with 90% contours for a choice of the primordial D abundance and two choices for the primordial ${}^4\text{He}$ abundance (see the text for details). Green contours are for deuterium abundances $(\text{D}/\text{H})_P$. Blue contours are for the ${}^4\text{He}$ mass fraction Y_P . The thick, black ellipse is the 90% confidence contour using the Peimbert et al. [38] helium-4 abundance, and the thin, grey error ellipse is the 90% confidence contour corresponding to the more conservative Steigman [19] helium-4 abundance. The purple contours are for the $(\text{Li}/\text{H})_P$ abundance; however, these were not used to constrain either χ or η .

dance determinations agree with the corresponding results of the BBN calculations at a given point in the $\{\eta, \chi\}$ parameter space. The thick black contour is for the 90% range in η_{10} and χ corresponding to the O'Meara et al. [26] deuterium abundance and the Peimbert et al. [38] helium abundance. The narrowness of this contour in the vertical (χ) direction is a direct consequence of the size of the Peimbert et al. [38] estimate of the error in Y_P . Given the sensitivity of χ to Y_P , it is interesting to explore the consequence of adopting a different central value and uncertainty in Y_P , while keeping the same primordial D abundance. To this end, we choose $Y_P = 0.240 \pm 0.006$ from [19]. The thin, gray contour in Fig. 1 corresponds to the 90% range for this alternate choice of Y_P . Both choices are consistent with the standard, Friedman-Lemaitre result $\chi = 1$.

III. CONCLUSIONS

As illustrated in Figure 1, the combined constraints are, within the uncertainties, consistent with the general relativity prediction of $\chi = 1$ and the independent (of BBN) WMAP constraint on η_{10} of 6.1 ± 0.2 [32] which corresponds to $\chi = 1$. For the Peimbert et al. [38] choice of Y_P , $\chi = 1.00 \pm 0.14$, while for the Steigman [19] helium abundance, $\chi = 0.84 \pm 0.25$. Note that the data strongly exclude $\chi = 0$. The current light element observations and BBN computations have provided a test of the general relativistic self-gravity of pressure. Since the modification of GR we are testing corresponds, for the radiation-dominated evolution appropriate for BBN, to an overall multiplicative factor of the product of Newton's gravitational constant and the radiation density, $G\rho \rightarrow G\rho \left(\frac{1+\chi}{2}\right)$, our result is equivalent to the BBN constraint on the variation of Newton's constant or, alternatively, to a modification of the radiation energy density as

parameterized by the effective number of neutrinos (see §1 for references).

$$\frac{1+\chi}{2} = 1 + \frac{\Delta G}{G} = 1 + \frac{7\Delta N_\nu}{43} \quad (12)$$

Assuming that these other parameters take on their standard-model values ($\Delta G = \Delta N_\nu = 0$), the self-gravity of the radiation (photons and neutrinos) pressure during the BBN epoch has been constrained quantitatively.

Acknowledgments

JS acknowledges support from the Paul E. Gray (1954) Endowed Fund for UROP. The research of GS is supported by a grant from the DOE. We thank Ed Bertschinger, James Felten, Alan Guth, Scott Hughes, and Alan Levine for extremely helpful discussions.

-
- [1] I. I. Shapiro, *Physical Review Letters* **13**, 789 (1964).
 - [2] B. Bertotti, L. Iess, and P. Tortora, *Nature (London)* **425**, 374 (2003).
 - [3] R. A. Hulse and J. H. Taylor, *Astrophys. J., Lett.* **195**, L51 (1975).
 - [4] J. H. Taylor, L. A. Fowler, and P. M. McCulloch, *Nature (London)* **277**, 437 (1979).
 - [5] J. M. Weisberg and J. H. Taylor, *Physical Review Letters* **52**, 1348 (1984).
 - [6] A. S. Bolton, S. Rappaport, and S. Burles, *Phys. Rev. D* **74**, 061501 (2006), arXiv:astro-ph/0607657.
 - [7] J. H. Taylor and J. M. Weisberg, *Astrophys. J.* **253**, 908 (1982).
 - [8] A. Friedmann, *Z. Phys* **10**, 377 (1922).
 - [9] H. P. Robertson, *Astrophys. J.* **82**, 284 (1935).
 - [10] A. G. Walker, *Proc. Lon. Math. Soc.* **42**, 90 (1936).
 - [11] G. H. Lemaitre, *Ann. Soc. Sci. Bruxelles* **A47**, 49 (1927).
 - [12] R. V. Wagoner, W. A. Fowler, and F. Hoyle, *Astrophys. J.* **148**, 3 (1967).
 - [13] L. Kawano, *FERMILAB-Pub-92/04-A* (1992).
 - [14] G. R. Caughlan and W. A. Fowler, *Atomic Data and Nuclear Data Tables* **40**, 283 (1988).
 - [15] P. Descouvemont, A. Adahchour, C. Angulo, A. Coc, and E. Vangioni-Flam, *Atomic Data and Nuclear Data Tables* **88**, 203 (2004).
 - [16] R. V. Wagoner, *Astrophys. J.* **179**, 343 (1973).
 - [17] J. Yang, D. N. Schramm, G. Steigman, and R. T. Rood, *Astrophys. J.* **227**, 697 (1979).
 - [18] J. P. Kneller and G. Steigman, *New Journal of Physics* **6**, 117 (2004), arXiv:astro-ph/0406320.
 - [19] G. Steigman, *Ann. Rev. Nucl. Part. Sci.* **57**, 463 (2007).
 - [20] G. Steigman, D. N. Schramm, and J. E. Gunn, *Physics Letters B* **66**, 202 (1977).
 - [21] L. Bergström, S. Iguri, and H. Rubinstein, *Phys. Rev. D* **60**, 045005 (1999).
 - [22] G. Steigman, *Nature (London)* **261**, 479 (1976).
 - [23] C. J. Copi, A. N. Davis, and L. M. Krauss, *Physical Review Letters* **92**, 171301 (2004), astro-ph/0311334.
 - [24] P. J. Kernan, G. D. Starkman, and T. Vachaspati, *Phys. Rev. D* **54**, 7207 (1996).
 - [25] J. Geiss and H. Reeves, *Astron. Astrophys.* **18**, 126 (1972).
 - [26] J. M. O'Meara, S. Burles, J. X. Prochaska, G. E. Prochter, R. A. Bernstein, and K. M. Burgess, *Astrophys. J., Lett.* **649**, L61 (2006), astro-ph/0608302.
 - [27] F. Hoyle and R. J. Tayler, *Nature (London)* **203**, 1108 (1964).
 - [28] Y. I. Izotov and T. X. Thuan, *Astrophys. J.* **602**, 200 (2004), arXiv:astro-ph/0310421.
 - [29] Y. I. Izotov, T. X. Thuan, and G. Stasińska, *Astrophys. J.* **662**, 15 (2007), arXiv:astro-ph/0702072.
 - [30] S. G. Ryan, T. C. Beers, K. A. Olive, B. D. Fields, and J. E. Norris, *Astrophys. J., Lett.* **530**, L57 (2000), arXiv:astro-ph/9905211.
 - [31] M. Asplund, D. L. Lambert, P. E. Nissen, F. Primas, and V. V. Smith, *Astrophys. J.* **644**, 229 (2006), arXiv:astro-ph/0510636.
 - [32] D. N. Spergel, R. Bean, O. Doré, M. R. Nolta, C. L. Bennett, J. Dunkley, G. Hinshaw, N. Jarosik, E. Komatsu, L. Page, et al., *Astrophys. J., Suppl. Ser.* **170**, 377 (2007), arXiv:astro-ph/0603449.
 - [33] E. Milne and W. McCrea, *QJ Maths* **5**, 73 (1934).
 - [34] J.-P. Uzan and R. Lehoucq, *European Journal of Physics* **22**, 371 (2001), arXiv:physics/0108066.
 - [35] W. Rindler, *Relativity: Special, General, and Cosmological* (Oxford University Press, 2001).
 - [36] S. Burles, K. M. Nollett, and M. S. Turner, *Phys. Rev. D* **63**, 063512 (2001).
 - [37] S. Burles, K. M. Nollett, and M. S. Turner, *Astrophys. J., Lett.* **552**, L1 (2001), astro-ph/0010171.
 - [38] M. Peimbert, V. Luridiana, and A. Peimbert, *Astrophys. J.* **666**, 636 (2007), arXiv:astro-ph/0701580.
 - [39] H. Reeves, J. Audouze, W. A. Fowler, and D. N. Schramm, *Astrophys. J.* **179**, 909 (1973).
 - [40] R. I. Epstein, J. Lattimer, and D. N. Schramm, *Nature (London)* **263**, 198 (1976).
 - [41] K. Davidson and T. D. Kinman, *Astrophys. J., Suppl. Ser.* **58**, 321 (1985).
 - [42] M. H. Pinsonneault, G. Steigman, T. P. Walker, and V. K. Narayanan, *Astrophys. J.* **574**, 398 (2002), astro-ph/0105439.
 - [43] A. J. Korn, F. Grundahl, O. Richard, P. S. Barklem, L. Mashonkina, R. Collet, N. Piskunov, and B. Gustafsson, *Nature (London)* **442**, 657 (2006), arXiv:astro-ph/0608201.
 - [44] P. Bonifacio, L. Pasquini, P. Molaro, E. Carretta, P. François, R. G. Gratton, G. James, L. Sbordone, F. Spite, and M. Zoccali, *Astronomy and Astrophysics* **470**, 153 (2007), arXiv:0704.2342.